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STUDY THE DIFFERENT FEEDBACK METHODS IN MANUFACTURING SETUP ERRORS FOR CONSTANT AND RANDOM VARIABLES

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ABSTRACT

The performance criteria is the quadratic off-target cost incurred over a small number of parts produced. Analytical formulae are presented and numerically illustrated. Two cases are considered, the first one where the setup error is constant but unknown offset and the second one where the setup error is a random variable with unknown first two moments. It is shown how Grubbs' harmonic rule and a simple integral provide a robust adjustment strategy in a variety of circumstances. As a by-product, the formulae presented in this paper allow to compute the expected off-target quadratic cost when a sudden shift occurs during production (not necessarily at setup) and the adjustment scheme compensates immediately after its occurrence.

1. INTRODUCTION

Suppose that due to a defective setup machine operation, the quality characteristic generated by a production process is in state of statistical control but the process starts off-target. Adjusting the process is justified if the only relevant cost is the cost of running the process off-target. Sequential adjustment rules for this problem were proposed by Grubbs (1983). These rules have been shown to derive from much broader class of setup adjustment problems based on stochastic control methods by Del Castillo, Pan and Colosimo (2002). In this paper, the small sample performance of Grubbs adjustment rules are studied and contrasted with other feedback adjustment methods.

To introduce some basic notation that will be based in that follows, let Y_t denote the observed derivation from target of some quality characteristic of interest, following Grubbs (1983), a simple but useful model for the setup adjustment problem is to assume

$$Y_t = d + U_{t-1} + \upsilon_t = \mu_t + \upsilon_t \qquad (1)$$

Where d is the setup error, U_t is the level of controllable factor set after producing part t (this will have an immediate effect on part t+1), μ_t is the main deviation from target for part t, and $\upsilon_t \sim N(0, \sigma_\upsilon^2)$ models both the part-to-part variability and the measurement error.

Two different control rules were derived by Grubbs depending on two sets of assumptions made on setup error d:

1. If d is an unknown constant, minimization of Var(μ_{n+1}) subject to E [μ_{n+1}]=0 results in Grubbs' "harmonic rule" :$U_t = -\hat{d}_t$,

$$\hat{d}_t=\hat{d}_{t-1}+K_t Y_t \quad (2)$$

$$K_t = 1/t$$

Or

$$U_t - U_{t-1} = \nabla U_t =- \frac{Y_t}{t}$$

Thus the weights K_t follow a harmonic series. An initial (a priori) estimate $\hat{d}_0$ is required to set the first setting of the controllable factor at $U_0 = -\hat{d}_0$. Del Castillo, Pan and Colosimo (2002) point out how adjustment rules (2) is a particular case of Robbins and Monro's (1951) celebrated stochastic approximation method. Therefore, this paper contains (indirectly) a small-sample performance study of stochastic approximation methods applied to the simple case of estimation of an unknown constant.

2. If $d \sim N(\bar{d},\sigma_v^2)$ the both $\bar{d}$ and σ_d^2 known, minimization of $E\,[\Sigma_{t=1}^{n}\mu_t^2]$ is achieved by Grubbs second adjustment rule. In this case,

3.

$$\hat{d}_t = \hat{d}_{t-1} + K_t Y_t\,,\ K_t = \frac{1}{t+\frac{\sigma_v^2}{\sigma_d^2}} \text{ and } \nabla U_t = - K_t Y_t \quad (3)$$

This rule is called Grubbs' "extended" rule by Trietsch (1998).

$$K_t =\frac{1}{t+\frac{\sigma_v^2}{\sigma_d^2}}, \text{ and } \nabla U_t = - K_t Y_t \quad (4)$$

The interpretation is that, a priori, $d \sim (\hat{d}_0, P_0)$. Thus for Grubbs' extended rule to be optimal with respect to $E\,[\Sigma_{t=1}^{n}\mu_t^2]$ we need to know $\bar{d}$ (so we can set $\bar{d}_0 = \bar{d}$) and σ_d^2 and σ_v^2 must be known for us to use (3). Evidently, if $P_0 = \sigma_d^2$, (3) and (4) are identical. Note how under this interpretation, if there is no prior information on offset ($P_0 \to \infty$) is equivalent to (2), Grubbs' simpler harmonic rule.

An additional adjustment rule that will be contrasted is an integral controller (Box and Luceno 1997);

$$\nabla U_t = - \lambda Y_t \quad (5)$$

Which, contrary to Grubbs' rules, does not converge to zero since λ is a constant.

2. PERFORMANCE INDICES FOR SS

The Performance indices that will be used in the reminder of this paper are presented in this section for the two cases considered by Grubbs.

Consider first the case where the setup error *d* is an unknown constant or "offset". For this case, performance index considered is the scaled Average Integrated Square Deviation (AISD) incurred over *m* time instants or parts. This is defined for integer $m > 0$ as:

$$(\mathrm{AISD}(m)) = \frac{1}{m\sigma_v^2}\sum_{i=1}^{m} E[Y_\mathrm{i}^2] = \frac{1}{m\sigma_v^2}\sum_{i=1}^{m}(\mathrm{V}[Y_i]\ E[Y_\mathrm{i}^2]) \qquad (6)$$

The AISD is a common performance index in the control engineering literature. Since Y_i models deviations from target, the AISD index is like average "variance plus squared bias" calculation, and a surrogate of a quadratic off-target "loss" function. We avoid dependency on σ_v^2 by dividing by this quantity.

$$\left(AISD_d(m)\right) = \frac{1}{m\sigma_v^2}E_d\sum_{i=1}^{m} E[Y_\mathrm{i}^2] = \frac{1}{m\sigma_v^2}\int_{-\infty}^{\infty}\sum_{i=1}^{m} E[Y_\mathrm{i}^2]\, f_d(x)dx \qquad (7)$$

Where the outer expectation is taken over the distribution of *d*. note that if *d* is a non-random constant, then AISD(m) = $\mathrm{AISD}_d(m)$. The case when *d* is normal with known mean and known variance was discussed by Trietsch (1998). Under such conditions, Gribbs' extended rule is optimal for the AISD_d criterian.

3. PERFORMANCE FOR AN UNKNOWN CONSTANT SETUP ERROR.

Suppose *d* is an unknown constant but unaware of this fact a user applies Grubbs' extended rule (i.e. the Kalman filter adjustment scheme given by (4)) To the process. It is shown in appendix A that this rule applied to the process $Y_\mathrm{t} = \mathrm{d} + U_{\mathrm{t}-1} + \upsilon_\mathrm{t}$ (with *d* constant) results in

$$\frac{E[Y\mathrm{t}]}{(\sigma_\upsilon)} = \frac{A}{B_1(t-1)+1} \qquad (8)$$

And

$$= 1\ \frac{t-1}{\left((\frac{1}{B_1}+t-1)^2\right)} \qquad (9)$$

Where, $A = (d - \widehat{d_0})/\sigma_v$ measures how far off the initial estimate of the offset was. The quantity $B_1 = P_0/{\sigma_v}^2$ is a measure of the "confidence" on the initial offset estimate.

To study the performance of Grubbs adjustment rules, equations (8) and (9) can be substituted into equation (6) and the sum computed for given values of A, B_1 and m. Note that our analytic expressions are exact and avoid use of simulation to estimate the AISD(m). Alternatively, an expression for the sum in AISD(m) is given by formula (17) in Appendix A which can be easier to use if a software that computes the polygamma function is available (e.g., Mathematica or Maple). We note that the corresponding expressions for Grubbs harmonic rule are obtained from (8) and (9) by letting $B_1 \rightarrow \infty$.

For a discrete integral controller (or EWMA controller), it can be shown that

$$\frac{E[Yt]}{(\sigma_v)} = ((1-\lambda)^{t-1} A) \qquad (10)$$

and

$$\frac{V[Yt]}{\sigma_v^2} = \frac{2-\lambda(1-\lambda)^{2(t-1)}}{2-\lambda} \qquad (11)$$

From where AISD(m) = $\frac{1}{m\sigma_v{}^2}\sum_{i=1}^{m}(E[Y_i]^2 + V[Y_i])$ can be computed or one can use the closed-form expression (eq. 18) in Appendix A. The AISD(m) expressions allow to study the trade-offs between the sum of the variances and the sum of squared expected deviations (squared bias). For the kalman filter scheme, as $B_1 = P_0/\sigma_v{}^2 \rightarrow 0$, implying increasingly higher confidence in the priori offset estimate, then $m^1 \sum_{i=1}^{m}(V[Y_i]/\sigma_v{}^2) \rightarrow 1$ (i.e., we get lower variance), but $m^1 \sum_{i=1}^{m}(E[Y_i]/\sigma_v{}^2) \rightarrow A^2$ (i.e., we get larger bias). Similarly, for the EWMA or integral controller, as $\lambda \rightarrow 0$, implying less weight given to the last observation we have $m^1 \sum_{i=1}^{m}(V[Y_i]/\sigma_v{}^2) \rightarrow 1$ (lower variance), but $m^1 \sum_{i=1}^{m}(E[Y_i]/\sigma_v{}^2) \rightarrow A^2$ (larger bias).

There are two parameters that can be modified in Grubbs adjustment rules: $\widehat{d_0}$ and P_0. The effect of these parameters can be studied from looking at the effect of changes in A and B_1, as previously defined. Therefore, the four scenarios presented in Table 1 were investigated.

	B_1 small	B_1 large
$\lvert A\rvert$ small	Good choice (case 1)	Bad choice (case 2)
$\lvert A\rvert$ large	Bad choice (case 3)	Good choice (4)

Table 1: Scenarios of interest, adjusting schemes, A = (d - $\widehat{d_0}$)/σ_v, $B_1 = P_0/\sigma_v{}^2$

In the table, if the initial prior variance P_0 is large relative to $\sigma_v{}^2$ (i.e., if B_1 is large), the weights K_t will be close to 1/t (Grubbs harmonic rule), i.e., the initial estimate $\widehat{d_0}$ will be discounted faster. This turns out to be a good decision if the initial offset estimate is far from d, where the distance between d and $\widehat{d_0}$ is measured relative to σ_v. A similar good decision is when P_0 is low and $\widehat{d_0}$ is a good estimate of offset (B_1 small, $|A|$ small). In such case, $K_t <$ 1/t, so there will be slower discounting of the initial estimate $\widehat{d_0}$. Cases (2) and (3) on the table indicate bad decisions, when the value of P_0 does not reflet how good the initial offset estimate really is. Since in the absence of historical information it is difficult to know a priori the value of practical interest to study the four cases on the Table.

Table 2 contrast the AISD performance of Grubbs harmonic rule, the discrete integral controller (EWMA controller) and the kalman filter adjusting scheme ($\sigma_v{}^2$ known). The table shows the values of AISD(m) for m = 5,10 and 20. As can be seen from the Table, the "gap" between the column minimum and the AISD provided by Grubbs rule shrinks as the offset d gets much larger than σ_v (i.e., as $|A|$ increases). This gap, however, is quite moderate except in the case where one is very confident

($B_1 = P_0/\sigma_v^{2}$ small) of our a priori offset estimate turns out to be quite accurate (i.e., A = 0). This is not a practical case because it implies we practically know the value of the offset d.

m = 5

B_1 :	|A| = 0	|A| = 1	|A| = 2	|A| = 3
1/90	**1.00023**	1.95791	4.83092	9.61929
0.5	1.09344	1.48656	5.66589	4.63144
1	1.16394	**1.45667**	5.33486	3.79844
2	1.24138	1.47815	**2.1884**	**7** 3.37233
90	1.41043	1.61046	2.21056	**3.21074**
Grubbs	1.41667	1.61667	2.21667	3.21667
I controller (λ = 0.1)	1.01655	1.70215	3.75895	7.18696
I controller (λ = 0.2)	1.05601	1.55191	3.03962	5.51914
I controller (λ = 0.3)	1.10922	1.49030	2.63354	4.53894

m = 10

B_1 :	|A| = 0	|A| = 1	|A| = 2	|A| = 3
1/90	**1.00049**	1.91004	4.63869	9.18644
0.5	1.09038	1.31359	1.98323	3.09930
1	1.13792	**1.29290**	1.75783	2.53271
2	1.18491	1.30578	**1.66840**	2.27276
90	1.27952	1.37954	1.67959	**2.17969**
Grubbs	1.28290	1.38290	1.68290	2.18290
I controller (λ = 0.1)	1.02830	1.49063	2.87761	5.18925
I controller (λ = 0.2)	1.08060	1.35518	2.17890	3.55178
I controller (λ = 0.3)	1.14190	1.33782	1.92558	2.90518

m = 20

B_1 :	|A| = 0	|A| = 1	|A| = 2	|A| = 3
1/90	**1.00090**	1.82739	4.30685	8.43929
0.5	1.07243	1.19211	1.55117	2.14960
1	1.10008	**1.17989**	1.41931	1.81895
2	1.12585	1.18691	**1.37009**	1.67539
90	1.17564	1.22565	1.37568	**1.62573**
Grubbs	1.17739	1.22739	1.37739	1.62739
I controller (λ = 0.1)	1.03899	1.29825	2.07606	3.37240
I controller (λ = 0.2)	1.09568	1.23455	1.65116	2.34552
I controller (λ = 0.3)	1.15917	1.25721	1.55133	2.04152

Table 2: Kalman Filter adjusting scheme (σ_v^{2} known), Grubbs harmonic rule and Integral controller AISD performance. A = (d - $\widehat{d_0}$)/σ_v, $B_1 = P_0/\sigma_v^{2}$. Bold number are minimum by column

m = 10, N = 100				
B_1 :	A = 0	A = 1	A = 2	A = 3
1/90	**1.0009**	1.8208	4.2807	8.3805
0.5	1.0760	1.1233	1.2653	1.5019
1	1.0948	**1.1177**	1.1865	1.3012
2	1.1082	1.1224	1.1648	1.2354
90	1.1277	1.1377	1.1677	1.2177
Grubbs	1.1182	1.1287	**1.1583**	**1.2083**
I controller (λ = 0.1)	1.0493	1.2050	1.6719	2.4502
I controller (λ = 0.2)	1.1130	1.1508	1.2643	1.4535
I controller (λ = 0.3)	1.1798	1.2001	1.2610	1.3626

Table 3: Kalman Filter adjusting scheme ($\sigma_v{}^2$ known), Grubbs harmonic rule and I-controller AISD (10,100) performance. 10 adjustments were made after which 90 additional parts were produced. A = (d - $\widehat{d_0}$)/σ_v, $B_1 = P_0/\sigma_v{}^2$. Bold number are minimum by column

The table, it appears the value $\lambda = 0.2$ provides a relatively good trade-off between fast return to target and inflation of variance if the process is really on-target (no offset).

Adjusting only the first few times

It could be argued that in practice, only the first few adjustments will be implemented after which no further adjustments are made to machine. The machine then runs at the final setting that resulted at the end of adjustment until completion of the batch of N parts. The performance of the adjustment rules should be investigated on this assumption instead. For this reason, let m denote the number of adjustments implemented in a batch of size N parts produced. For discrete m such that 1 ≤ m ≤ n, the average integrated squared deviation index is defined as:

$$\text{AISD}(m,N) = \frac{mAISD(m)}{N} + \frac{(N-m)(VY_{m+1}+E[Y_{m+1}]^2)}{N\sigma_v^2}$$

Closed-form expressions for AISD(m,N) for the Kalman filter. Grubbs harmonic, and discrete integral control rules can be found in Appendix A. They were used to produce Table 3 where the AISD (10,100) performance indices were computed. Clearly, if m = N, then AISD(m,N) = AISD(m).

From Table 3 and similar computations for other values of m and N, it was observed that the conclusions expressed before based on the AISD(m) index, which measurs off-target cost only while the adjustments take place, are practically unchanged if we consider in addition the cost incurred after adjustments stop and the process keeps operating. Only when A → 0 (no offset or perfect offset estimate) and $B_1 = P_0/\sigma_v{}^2$ is small, the Kalman filter rule outperforms the harmonic rule. The discrete integral controller with $\lambda = 0.2$ is a good intermediate value that balances a rapid return to target for large offsets with a low inflation in variance in case the process was really on-target but we nevertheless adjust.

4. PERFORMANCE WHEN THE SETUP ERROR IS A RANDOM VARIABLE

Suppose now that the offset d is a random variable such that d ~ $(d_0 \sigma_d{}^2)$. Note that no assumption on the distribution on d is made. We wish to evaluate the performance of the different adjustment methods by averaging over the possible realizations of the random offset d.

As mentioned earlier, if the mean and variance of d are known, then the Kalman Filter scheme, and hence, Grubbs extended rule are optimum for a quadratic loss function such as our AISD_d criterian. This was the case discussed by Grubbs (1983) and Trietsch (1998). In this section we consider the more general case when mean and variance of d are both unknown.

When d is random, we need to use a prior estimate $\widehat{d_0}$ with associated variance P_0 to start the Kalman filter scheme (4). The situation is depicted in Figure 1. Using (7) as our performance index, it is shown in Appendix B that for the Kalman Filter (KF) scheme.

$$\text{AISD}_d(\text{m})_{KF} = C_1\left(B_2 + A_2{}^2\right) + C_2 \qquad (12)$$

Where A_2 = (d - $\widehat{d_0}$)/σ_v is a measure of the average error in the offset estimate, $B_2 = \frac{\sigma_d{}^2}{\sigma_v{}^2}$ is a measure of the variability of the setup, and C_1, and C_2 (and C_3 used next) are functions of $B_1 = P_0/\sigma_v{}^2$ and m shows in Appendix A. For Grubbs (G) harmonic rule this reduces to:

$$\text{AISD}_d(\text{m})_G = \frac{B_2 + A_2{}^2}{m} + C_3 \qquad (13)$$

Recall that B_1 is a measure of confidence in $\widehat{d_0}$, therefore, since $\sigma_d{}^2$ is not known, we have that in general $B_1 \neq B_2$. Closed-form expressions can be obtained for $\text{AISD}_d(\text{m})_{EWMA}$.

5. CONCLUSIONS AND FURTHER RESEARCH

In the setup error is an unknown constant it was shown that for most practical case when sequential adjustment is necessary, Grubbs (1983) harmonic rule represents a better strategy than the Kalman filter scheme (and therefore it performs better than Grubbs' extended rule). The even simpler integral or EWMA controller with height λ = 0.2 provides a competitive alternative to the harmonic rule for cases when the offset is small (in order of less than one standard deviation of the process). It was shown that these conclusions remain essentially unchanged if the performance is evaluated based not only while adjustments take place but also by considering additional runs in which no further adjustments are made to the process.

If the setup error is instead a random variable, an integral controller performs better than the Kalman filter scheme when setup noise is variable high and offset is very large on average. When the offset is large and/or the setup noise is large, Grubbs harmonic rule outperforms the Kalman filter scheme. The analytical formulae in Appendix B allow to obtain similar results for other values of the process and controller parameters without recourse to simulation. Further recommendations about when to use each method in the random setup error case can be reached by looking at figure 2-5.

The main advantage of the discrete integral controller over the other methods considered in this paper is that it stays alert for compensating for further shifts in this process that occur while manufacturing (i.e. not at setup). The weights in Grubbs rule, for example, would have to be reset every time a shift is detected. This points toward integration of the adjustment schemes with a detection mechanisms that will trigger the adaptation of the weights, along the lines recently followed by Guo et al. (2000). Such integrated SPC/EPC approaches require further research. The important connection between Grubbs harmonic rule and Robbins and Monro's stochastic Approximation method (1951) point out to a very large body of literature and theoretical results some of which may prove useful in process adjustment applications.

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