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3D ANALYSIS OF STRESS CONCENTRATION FACTOR AND DEFLECTION IN THIN ISOTROPIC AND ORTHOTROPIC PLATES WITH CENTRAL CIRCULAR HOLE SUBJECTED TO TRANSVERSE LOADING

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ABSTRACT

A number of analytical and numerical techniques are available for the two dimensional study of stress concentration around the hole(s) in isotropic and composite plates subjected to in-plane or transverse loading conditions. The information on the techniques for three dimensional analyses of stress concentration around the hole in isotropic and composite plates subjected to transverse loading conditions is, however, limited. In the present work, distributions of stresses and deflection in simply supported rectangular isotropic and orthotropic composite plates with central circular hole subjected to transverse static loading have been studied using 3-D finite element method. The effect of t/A and D/A ratio upon stress concentration factor and deflection in isotropic and orthotropic plates under transverse static loading condition is studied. The 3-D results are compared with 2-D results and it is observed that the 2-D results for orthotropic composite plates subjected to transverse loading conditions vary too much and hence should not be taken for conclusion. All results are presented in graphical form and discussed. The finite element formulation and its analysis are carried out using ANSYS package.

Keywords: Finite Element Analysis, Stress Concentration Factor, Deflection, Plate, Composite, Elastic Constants

1. INTRODUCTION

Isotropic, orthotropic and laminated composite plates with central circular hole under transverse loading, have found widespread applications in various fields of engineering such as aerospace, marine, automobile and mechanical. For design of such plates with hole, accurate knowledge of deflection, stresses and stress concentration factor are required. Stress concentration arises from any abrupt change in geometry of plate under loading; as a result, stress distribution is not uniform throughout the cross section. Failures such as fatigue crack and plastic deformation frequently occur at the points of stress concentration. Various researchers analyzed different cases of stress concentration in plate with circular holes.

Chaudhuri [1] worked on stress concentration around a part through hole weakening a laminated plate using finite element method. Paul and Rao [2, 3, 4] evaluated stress and stress concentration in fibre reinforced composite laminated plate containing central circular hole and two coaxial holes subjected to transverse load by using finite element method using Lo Christensen Wu higher order bending theory. Xiwu et al. [5, 6] studied a finite composite plate weakened by elliptical holes under different in-plane loading, treated as an anisotropic multiple connected plates, based on the classical plate theory. Using the complex potential method in the plane theory of elasticity of an isotropic body, an analytical solution concerned with stress concentration around an elliptical hole or holes in finite composite laminated plate is obtained. Ting et al. [7, 8] presented the alternating method to study the stress distributions of the multiple circular or multiple elliptical holes with the rhombic pattern in the infinite domain. Ukadgaonker and Rao [9] proposed a general solution for stresses around hole in symmetric laminates under in-plane loading by introducing a general form of mapping function and an arbitrary biaxial loading condition to the boundary conditions, and the basic formulation is extended for multilayered plates. Troyani et al. [10] have determined the in-plane theoretical stress concentration factors for short rectangular plates with centered circular holes subjected to uniform tension using finite element method. Kotousov and Wang [11] have presented analytical solutions for the three dimensional stress distributions around typical stress concentrators in an isotropic plate of arbitrary thickness based on the assumption of a generalized plane strain theory. Toubal et al. [12] studied stress concentration in a circular hole in composite plate. Jain and Mittal [13] have analyzed the stress concentration and deflection in isotropic, orthotropic and laminated composite plates with central circular hole subjected to transverse static loading by using two dimensional finite element method.

Work done by the various researchers with different approximation are excellent for some problems of stress concentration, but the literature available for the analysis of all cases of stress concentration around circular hole in isotropic or composite plates subjected to transverse loading conditions is limited. Hence, it needs attention for the analysis of more cases of stress concentration in isotropic and composite plates with hole subjected to transverse loadings.

The present work aims to study the behavior of SCF in isotropic and orthotropic composite plates with central circular hole subjected to transverse static loading for different geometry of plate and, also parallel study of deflection in all cases undertaken. The effect of *t/A* and *D/A* ratio on SCF for σ_x, σ_y, τ_{xy}, σ_{eqv} and, on U_z is investigated by using three dimensional finite element analysis in simply supported thin rectangular isotropic and orthotropic composite plates with central circular hole. The deflection values for different cases with and without hole are also compared. The 3-D finite element results are also compared with 2-D finite element results for thin plates. Results are obtained for three different orthotropic materials to find out the sensitivity of stress concentration factor and deflection on elastic constants also. The analytical treatment for such type of problem is difficult and hence the finite element method is adopted for whole analysis.

2. FORMULATION OF THE PROBLEM

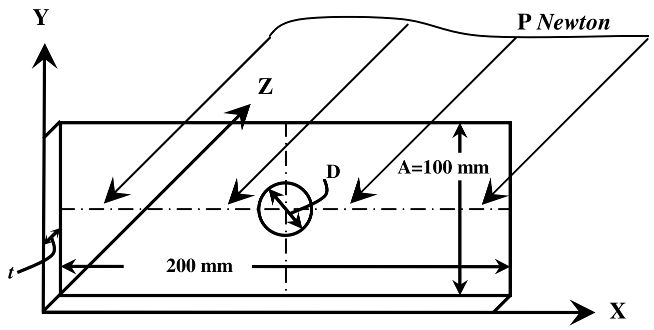


Figure 1 Details of model analyzed in study

An isotropic/orthotropic plate of dimension 200 *mm* X 100 *mm* with *t* thickness having a central circular hole of diameter *D* under uniformly distributed loading of *P* in transverse direction (Fig. 1), is taken for analysis. The material properties of isotropic material are selected as: [E_x, ν_{xy}]: [39 GPa, 0.3]. Three different composite materials are selected for the analysis of orthotropic composite plates. The material properties for the composite materials, selected for analysis are shown in Table 1.

Table 1 Material properties of composite materials [14]

Properties	Materials		
	E-glass/ epoxy	Boron/ epoxy	Boron/ aluminum
E_x	39 GPa	201 GPa	235 GPa
E_y	8.6 GPa	21.7 GPa	137 GPa
E_z	8.6 GPa	21.7 GPa	137 GPa
G_{xy}	3.8 GPa	5.4 GPa	47 GPa
G_{yz}	3.8 GPa	5.4 GPa	47 GPa
G_{zx}	3.8 GPa	5.4 GPa	47 GPa
ν_{xy}	0.28	0.17	0.3
ν_{yz}	0.28	0.17	0.3
ν_{zx}	0.28	0.17	0.3

3. FINITE ELEMENT ANALYSIS

An eight nodded Structural 3-D Shell Element (specified as, Shell93 in ANSYS package) with element length of 2 *mm* for isotropic and orthotropic plates with element length of 2 *mm* was selected for two dimensional finite element analyses. Each node has six degrees of freedom, making a total 48 degrees of freedom per element. A twenty nodded Structural 3-D Solid Element (specified as, Solid95 in ANSYS package) with element length of 2 *mm* for isotropic and orthotropic plates was selected for three dimensional finite element analyses. Each node has three degrees of freedom, making a total 60 degrees of freedom per element. Mapped meshing is used for all models so that more elements are employed near the hole boundary.

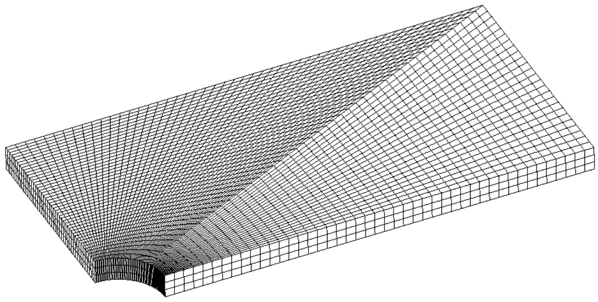


Figure 2 Typical example of finite element mesh for *D/A*=0.2 and *t/A*=0.05

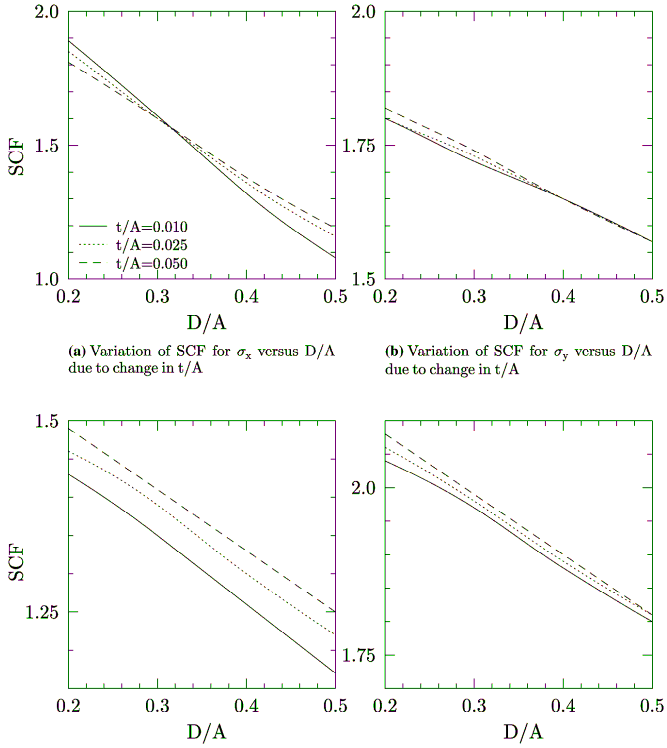
Due to the symmetric nature of different models investigated, the quadrant plate is discretized for finite element analysis. Numbers of checks and convergence tests are made for selection of suitable elements from different available elements and to decide the element length, for both 2D and 3D finite element analysis. Results were then displayed by using post processor of ANSYS programme. The discretized three dimensional finite element models for *D/A* =0.2 and *t/A*=0.05, used in study shown in figure, Figure 2 as an example.

4. RESULTS AND DISCUSSION

Numerical results obtained from 3D finite element analysis for thin simply supported isotropic and orthotropic plates with central circular hole subjected to uniformly distributed load, P in transverse direction are shown in figures, Fig. 3 to Fig. 7. The results are discussed and observations are recorded sequentially.

4.1. SCF in isotropic plate

The effect of *D/A* and *t/A* ratio on SCF (for σ_x, σ_y, τ_{xy} and σ_{eqv}) in isotropic plate subjected to uniformly distributed loading are shown in Figure 3.



(a) Variation of SCF for σ_x versus D/A due to change in t/A

(b) Variation of SCF for σ_y versus D/A due to change in t/A

(c) Variation of SCF for τ_{xy} versus D/A due to change in t/A

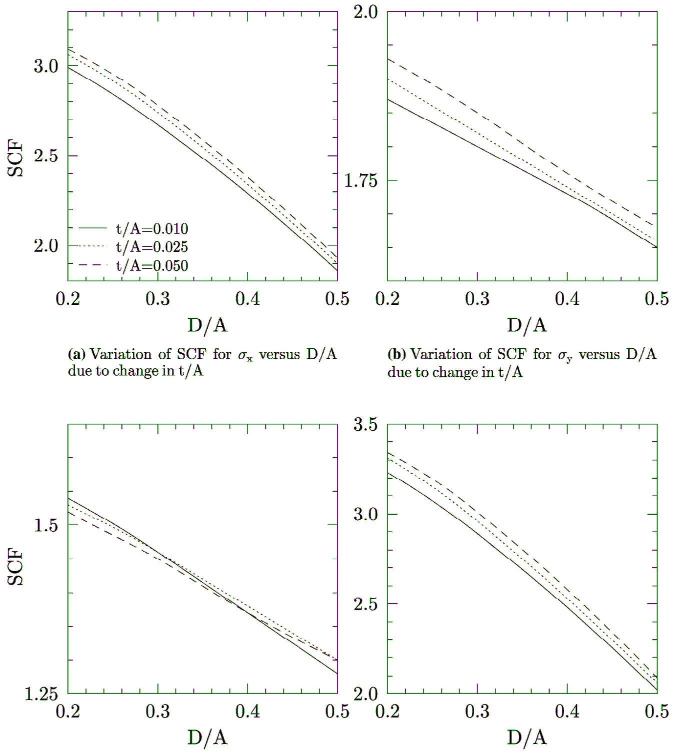
(d) Variation of SCF for σ_{eqv} versus D/A due to change in t/A

Figure 3 Variation of SCF in isotropic plate

Following observations can be made. A huge variation is not observed in SCF for the stresses with *t/A* for all *D/A* ratios. SCF for σ_y, τ_{xy} and σ_{eqv} increased with increase of *t/A* for all *D/A* ratio, however in case of σ_x, it initially decreased with increase of *t/A*, when *D/A* ratio is increased from 0.2 to 0.3 and then increased with increase of *t/A*, when *D/A* is ratio increased from 0.3 to 0.5. In all cases of *t/A*, it has been seen that SCF for all stresses continuously decreased with increase of *D/A* ratio. It has been also seen that the stress concentration for σ_x is significant for *D/A*=0.2, and negligible for *D/A*=0.5. It has been also observed that the variation of SCF with *D/A* is huge in case of σ_x and significant in case of σ_y, τ_{xy} and σ_{eqv}.

4.2. SCF in composite plates

The effect of *D/A* and *t/A* ratio on SCF (for σ_x, σ_y, τ_{xy} and σ_{eqv}) in e-glass/epoxy orthotropic composite plate subjected to uniformly distributed loading are shown in Fig. 4.



(a) Variation of SCF for σ_x versus D/A due to change in t/A

(b) Variation of SCF for σ_y versus D/A due to change in t/A

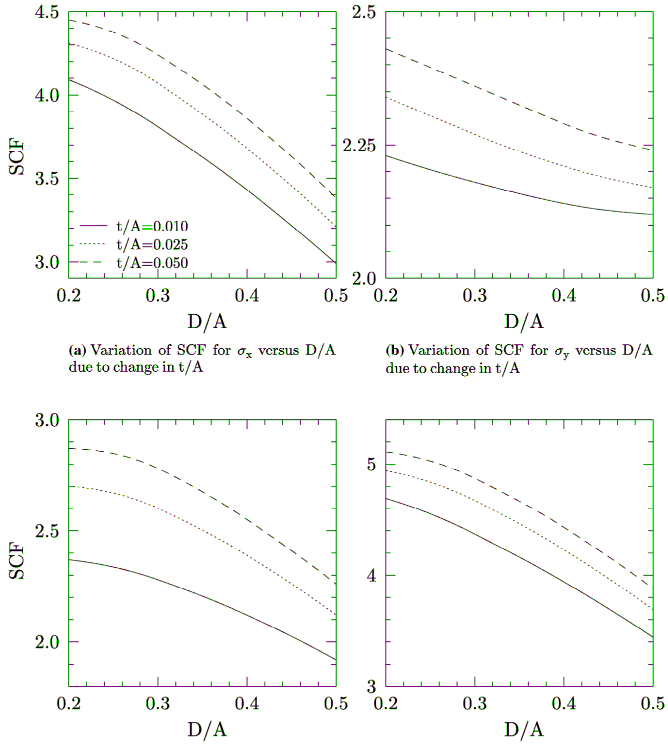
(c) Variation of SCF for τ_{xy} versus D/A due to change in t/A

(d) Variation of SCF for σ_{eqv} versus D/A due to change in t/A

Figure 4 Variation of SCF in e-glass/epoxy composite plate

Following observations can be made. A little variation is observed in SCF for all stresses with *t/A* for all *D/A* ratios. SCF for σ_x, σ_y and σ_{eqv} uniformly increased with increase of *t/A* for all *D/A* ratio, however in case of τ_{xy}, the variation is not uniform. In all cases of *t/A*, it has been seen that SCF for all stresses continuously decreased with increase of *D/A* ratio. It has been also observed that the variation of SCF with *D/A* is huge in case of σ_x and σ_{eqv} and, significant in case of σ_y and τ_{xy}.

The effect of *D/A* and *t/A* ratio on SCF (for σ_x, σ_y, τ_{xy} and σ_{eqv}) in boron/epoxy orthotropic composite plate subjected to uniformly distributed loading are shown in Fig. 5.

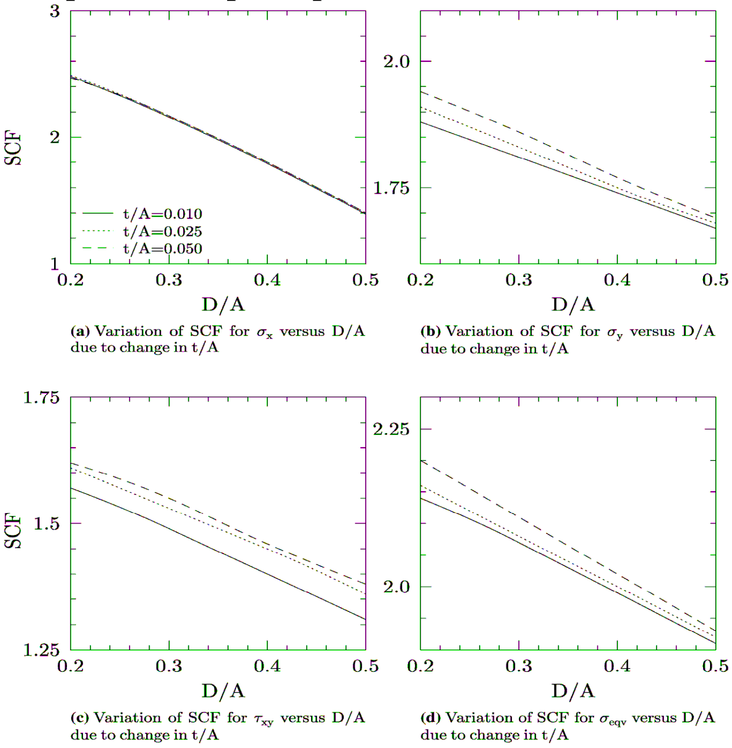


(a) Variation of SCF for σ_x versus D/A due to change in t/A
(b) Variation of SCF for σ_y versus D/A due to change in t/A
(c) Variation of SCF for τ_{xy} versus D/A due to change in t/A
(d) Variation of SCF for σ_{eqv} versus D/A due to change in t/A

Figure 5 Variation of SCF in boron/epoxy composite plate

Following observations can be made. A significant variation is observed in SCF for all stresses with *t/A* for all *D/A* ratios. SCF for all stresses increased with increase of *t/A* for all *D/A* ratio.
In all cases of *t/A*, it has been seen that SCF for all stresses continuously decreased with increase of *D/A* ratio. It has been also observed that the variation of SCF with *D/A* is huge in case of σ_x and σ_{eqv} and, significant in case of σ_y and τ_{xy}.
The effect of *D/A* and *t/A* ratio on SCF (for σ_x, σ_y, τ_{xy} and σ_{eqv}) in boron/aluminum orthotropic composite plate subjected to uniformly distributed loading are shown in Fig. 6. Following observations can be made.
A little variation is observed in SCF for σ_y, τ_{xy} and σ_{eqv} with *t/A* for all *D/A* ratios, however in case of σ_x, it is almost negligible. SCF for σ_y, τ_{xy} and σ_{eqv} uniformly increased slightly with increase of *t/A* for all *D/A* ratio. In all cases of *t/A*, it has been seen that SCF for all stresses continuously decreased with increase of *D/A* ratio.

4.3. Deflection in isotropic and composite plates



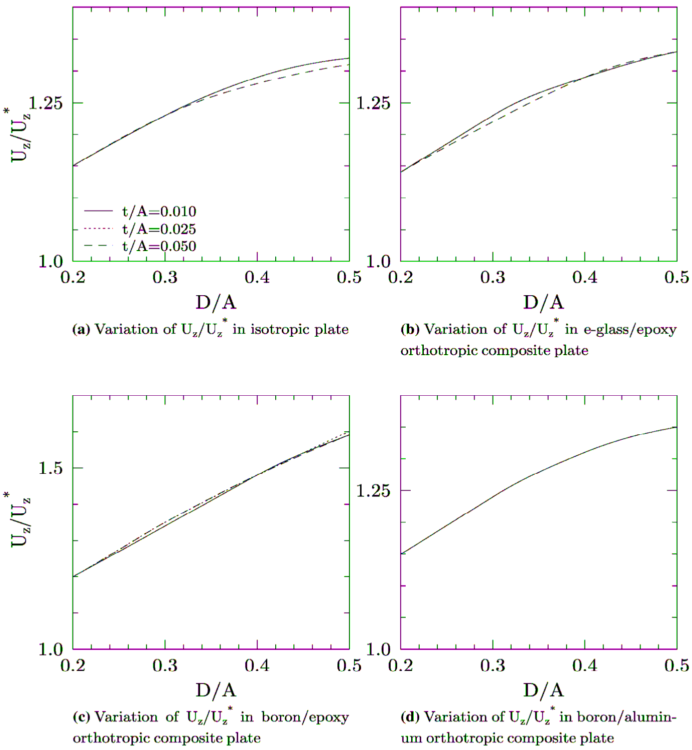
(a) Variation of SCF for σ_x versus D/A due to change in t/A

(b) Variation of SCF for σ_y versus D/A due to change in t/A

(c) Variation of SCF for τ_{xy} versus D/A due to change in t/A

(d) Variation of SCF for σ_{eqv} versus D/A due to change in t/A

Figure 6 Variation of SCF in boron/aluminum composite plate



(a) Variation of U_z/U_z^* in isotropic plate

(b) Variation of U_z/U_z^* in e-glass/epoxy orthotropic composite plate

(c) Variation of U_z/U_z^* in boron/epoxy orthotropic composite plate

(d) Variation of U_z/U_z^* in boron/aluminum orthotropic composite plate

Figure 7 Variation of U_z/U_z^* versus D/A ratio due to change in t/A ratio

The effect of *D/A* and *t/A* ratio on U_z/U_z^* in isotropic and orthotropic plates subjected to uniformly distributed loading are shown in Fig. 7.
Following observations can be made. Variation of U_z/U_z^* with *t/A* is almost negligible for all *D/A* ratios in isotropic and orthotropic plates both. The U_z/U_z^* continuously increased with increase of *D/A* ratio for isotropic and orthotropic plates both. It has been also observed that the variation of U_z/U_z^* with D/A ratio is more in composite plates in compare to isotropic plate.

4.4. Assessment of 2D and 3D results

The % variation in 2D results with 3D results of SCF (for σ_x, σ_y and σ_{eqv}) versus *D/A* ratio for *t/A*=0.01 in isotropic and orthotropic plates are shown in Fig. 8.

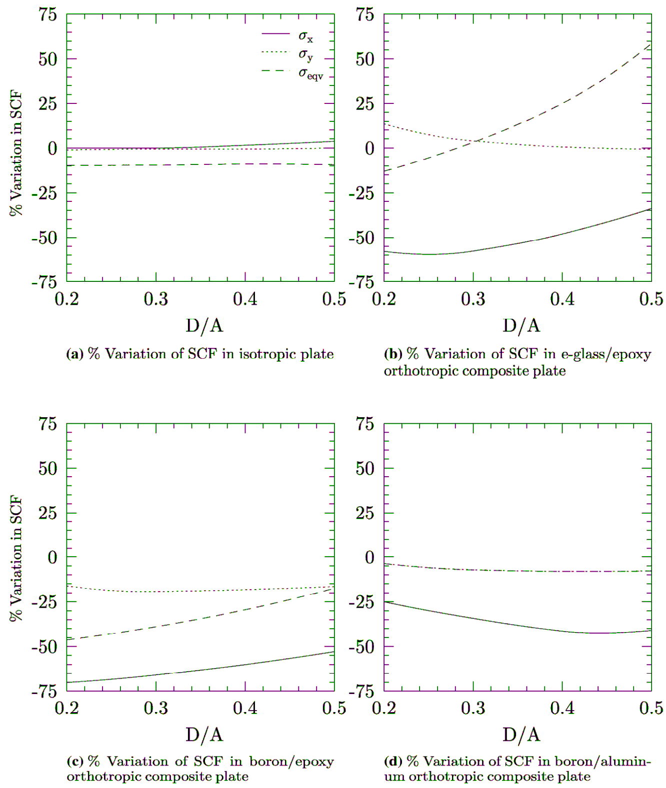
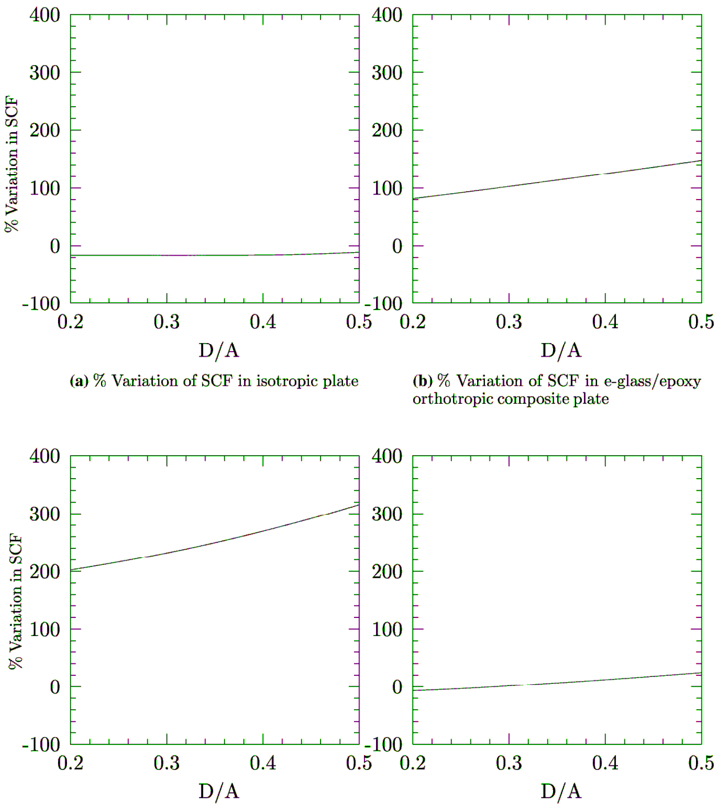


Figure 8 % Variation in 2D results with 3D results of SCF (for σ_x, σ_y and σ_{eqv}) versus D/A ratio for t/A=0.01

Following observations can be made. The % Variation in SCF values for σ_x, σ_y and σ_{eqv} of 2D and 3D results are much more in orthotropic plates as compared to isotropic plate. In case of isotropic plate, it is almost constant with *D/A* ratio and varied within the range of -10.53 to 3.70 %.
In case of e-glass/epoxy orthotropic composite plate; when *D/A* ratio increase from 0.2 to 0.5, 2D values of SCF are varied with 3D values within the range of –57.68 to -33.87 %, -0.61 to 13.37 % and -13.00 to 58.42 % for σ_x, σ_y and σ_{eqv} respectively.
In case of boron /epoxy orthotropic composite plate; when *D/A* ratio increase from 0.2 to 0.5, 2D values of SCF are varied with 3D values within the range of –70.17 to -52.84 %, -19.27 to -16.14 % and -46.27 to -17.44 % for σ_x, σ_y and σ_{eqv} respectively.

In case of boron /aluminum orthotropic composite plate; when *D/A* ratio increase from 0.2 to 0.5, 2D values of SCF are varied with 3D values within the range of –41.34 to -25.00 %, -8.05 to -3.72 % and -8.04 to -3.74 % for σ_x, σ_y and σ_{eqv} respectively.
It has been observed that % variation in 2D results with 3D results of SCF for σ_y versus *D/A* ratio is almost constant for all composite materials. It has been also seen that magnitude of % variation in 2D results with 3D results of SCF is maximum for σ_x and minimum for σ_y for all *D/A* ratio and composite plate.
The % variation in 2D results with 3D results of SCF for τ_{xy} versus *D/A* ratio for *t/A*=0.01 in isotropic and orthotropic plates are shown in Fig. 9. Following observations can be made.



(a) % Variation of SCF in isotropic plate
(b) % Variation of SCF in e-glass/epoxy orthotropic composite plate
(c) % Variation of SCF in boron/epoxy orthotropic composite plate
(d) % Variation of SCF in boron/aluminum orthotropic composite plate

Figure 9 % Variation in 2D results with 3D results of SCF for τ_{xy} versus D/A ratio for t/A=0.01

The % Variation in SCF values for τ_{xy} of 2D and 3D results are much more in orthotropic plates as compared to isotropic plate. In case of isotropic plate, it is almost constant with *D/A* ratio and varied within the range of -16.08 to -11.11 %.
In case of e-glass/epoxy orthotropic plate, 2D value of SCF for τ_{xy} is varied with 3D value within the range of 81.82 to 147.66 % for all *D/A* ratios.
In case of boron/epoxy orthotropic plate, 2D value of SCF for τ_{xy} is varied with 3D value within the range of 202.53 to 315.63 % for all *D/A* ratios.
In case of boron/aluminum orthotropic plate, 2D value of SCF for τ_{xy} is varied with 3D value within the range of -7.01 to 24.43 % for all *D/A* ratios.

It has been observed that magnitude of % variation in 2D results with 3D results of SCF for τ_{xy} is increased with increase of *D/A* ratio for all composite materials.
The % variation in 2D results with 3D results of U_z/U_z^* versus *D/A* ratio for *t/A*=0.01 in isotropic and orthotropic plates are shown in Fig. 10. Following observations can be made**.**

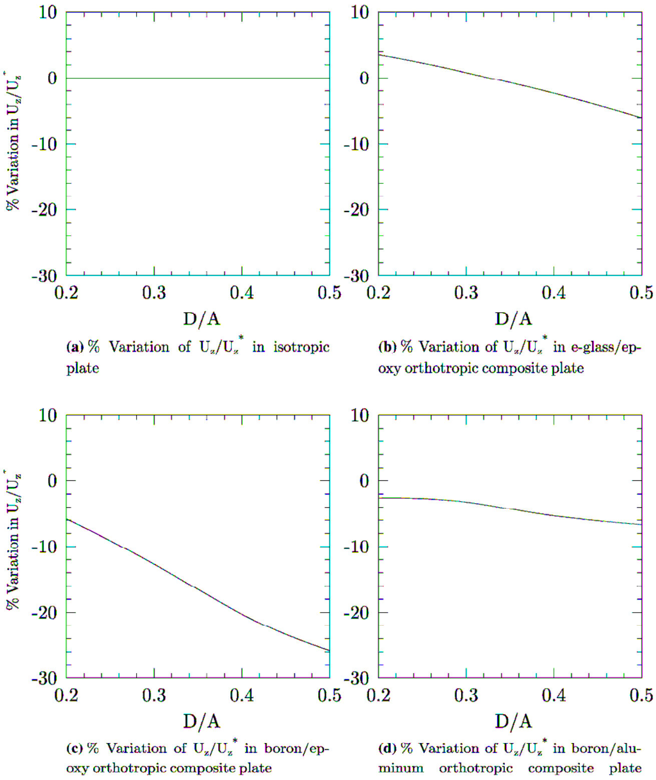


Figure 10 % Variation in 2D results with 3D results of U_z/U_z^* versus D/A ratio for t/A=0.01

The % Variation in SCF values for U_z/U_z^* of 2D and 3D results are much more in orthotropic plates as compared to isotropic plate. However, in case of isotropic plate, it is almost negligible.
In case of e-glass/epoxy orthotropic plate, 2D value of U_z/U_z^* is varied with 3D value within the range of -6.02 to 3.51 % for all *D/A* ratios.
In case of boron/epoxy orthotropic plate, 2D value of U_z/U_z^* is varied with 3D value within the range of -25.79 to -5.83 % for all *D/A* ratios.
In case of boron/aluminum orthotropic plate, 2D value of U_z/U_z^* is varied with 3D value within the range of -6.67 to -2.61 % for all *D/A* ratios.
It has been observed that magnitude of % variation in 2D results with 3D results of U_z/U_z^* is increased with increase of *D/A* ratio.

5. CONCLUSION

On the basis of results presented above, following can be concluded. In general, the maximum stress concentration always occurs on hole boundary in case of isotropic and orthotropic composite simply supported plates both. The SCF for all stresses plays an important role in all cases. The stress concentration for all stresses is higher in orthotropic composite plates then

isotropic plate. It has been seen that SCF for all stresses decreases with increase of *D/A* ratio for all cases. It has been also observed that SCF for all stresses increased slightly with increase of *t/A* at any D/A ratio for almost all cases of materials. The *t/A* ratio plays a significant role in orthotropic plates and negligible role in isotropic plate. In case of isotropic and orthotropic plates both, the U_z/U_z^* continuously increased with increase of *D/A* ratio. The effect of *t/A* is almost negligible on U_z/U_z^* in isotropic and orthotropic plates both. Effect of *D/A* ratio on U_z/U_z^* is obtained more in orthotropic plates as compare to isotropic plate.
In case of orthotropic plates the variation of SCF and U_z/U_z^* with *D/A* and *t/A* ratio depends on elastic constants of material. The results of work reveal that the SCF highly depend on the ratio of E_x/G_{xy} and E_y/G_{xy}. The results obtained show that for higher values of these ratios, SCF is also higher.
The 3-D finite element analysis results are also accessed with 2-D finite element analysis results. 2-D results of SCF for all stresses are quite good in case of isotropic plate, but in case of orthotropic plates, 2-D values of SCF for all stresses are not accurate. SCF for τ_{xy} gives a distorted picture when calculated from 2-D analysis, since 2-D results have variation up to 300 % from 3-D results. So it can be concluded that the 2-D results for SCF should not be used for orthotropic plates subjected to transverse loading conditions. 2-D analysis can be used for isotropic materials, and for orthotropic materials having low ratio of E_x/G_{xy} and E_y/G_{xy} such as boron/aluminum material. In case of U_z/U_z^*, the variation in 2-D and 3-D results is very small for all the cases. The variation in case of isotropic plate is almost negligible; hence 2-D analysis can be used for calculation of deflection in transverse direction for isotropic plate.

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Nomenclature

A	width of plate (mm)
D	diameter of hole (mm)
E_i	modulus of elasticity for i direction
G_{ij}	modulus of rigidity for ij plane
P	uniform distributed load (N)
SCF	stress concentration factor
t	thickness of plate
U_z	deflection in transverse direction in plate with hole
U_z^*	deflection in transverse direction in solid plate
σ_i	normal stress in i direction
σ_{eqv}	von mises (equivalent) stress
τ_{xy}	shear stress in ij plane
ν_{ij}	Poisson's ratio in ij plane